Managing a Crypto-Currency Portfolio via MinMax Drawdown Control

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Abstract

Crypto-currencies and other innovative asset classes present a fundamental challenge for quantitative asset-allocation. Because the track record of innovative assets is by definition short, it is difficult to form reliable estimates of expected returns and covariance matrices needed as inputs for standard portfolio optimization. Even if such estimates are available, they may be useless to investors if the behavior of underlying assets changes over time. Building on the MinMax Drawdown Control framework of Chassang (2018), this paper proposes a conceptually attractive and empirically successful approach to build benchmark portfolios of crypto-currencies and other innovative assets.

REFERENCES: crypto-currencies, MinMax Drawdown Control, prior-free asset allocation, agnostic asset allocation, innovative assets.

1 Introduction

Black and Litterman (1992) uncover a central challenge for portfolio allocation. Natural implementations of mean-variance optimal portfolios (Markowitz, 1952), using historical data to estimate returns and correlations, frequently lead to unattractive, leveraged, and highly

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unstable portfolios. One response to this issue has been to avoid including estimated returns as an input to portfolio construction: risk-parity (Lörtscher, 1990; Kessler and Schwarz, 1990; Sharpe, 2002) allocates portfolio weights on the basis of volatility estimates alone; DeMiguel et al. (2009) defends the merits of equal-weight portfolios. The limitations of historical data as a determinant of portfolio allocation seems particularly salient in the case of innovative asset classes with limited track record.

Chassang (2018) tackles the problem of portfolio allocation over novel or changing assets and argues the merits of worst case drawdown guarantees as a benchmark objective for dynamic asset allocation: a good dynamic asset allocation framework should guarantee low drawdowns relative to both the safe asset, and underlying risky assets. MinMax Drawdown Control helps an agnostic investor achieve low drawdowns for every possible realization of returns. This paper expands on Chassang (2018) in three ways.

(i) It formalizes the point that investors who have correct beliefs about the stochastic process for returns should experience low drawdowns with high probability;

(ii) It shows how to build portfolio indices that guarantee low drawdowns for overlapping generations of investors;

(iii) It illustrates the empirical value of MinMax Drawdown Control by applying it to the problem of managing a portfolio of crypto-currencies.

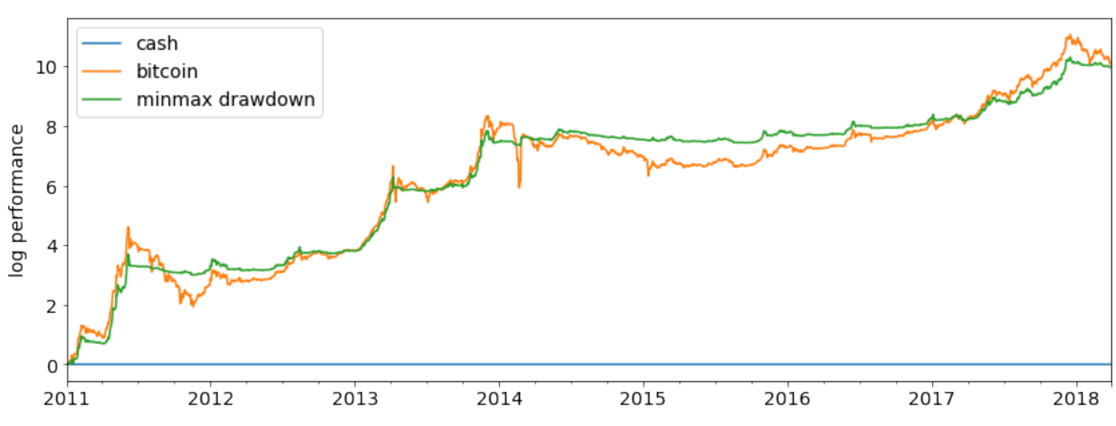


Figure 1: applying minmax drawdown control to cash and bitcoin.

Crypto-currencies are an interesting case-study for the MinMax Drawdown approach for several reasons. First, as Figure 1 illustrates, MinMax Drawdown Control considerably improves the risk-reward trade-off of a crypto-currency investor. Second, crypto-currencies have a short track-record available, making estimation difficult. Third, the asset class has experienced, and will mostly likely continue to experience, considerable changes, reflecting evolutions in the set of investors, regulation, and technological use cases.

2 Investing With and Without Priors

2.1 Setup

An investor with finite horizon $N \in \mathbb{N}$ allocates resources across several risky assets $i \in \{1, \cdots, I\}$ and a single risk-free asset denoted by 0.

For simplicity, returns $r^0 \geq 0$ to the risk-free asset are constant over time. Risky returns $(r^i)_{i\in\{1,\cdots,I\}}$ belong to a set M of market returns taking the form $M = [-\overline{r}, \overline{r}]^I$, with $\overline{r} \in (0, 1)$. Let $r_t \equiv (r^0, r^i_t)_{i\in\{1,\cdots,I\}}$ denote realized returns at time t.

Let $(a^i_t)_{i\in\{1,\cdots,I\}}$ and a^0_t respectively denote allocations to the risky and risk-free assets at time t. Allocations to risky assets must belong to $[\underline{a}, \overline{a}]^I$. In addition total allocation weights must sum to one, so that $a^0 = 1 - \sum_{i\in\mathcal{I}} a^i$. The overall allocation is denoted by $a = (a^0, a^i)_{i\in\{1,\cdots,I\}}$. We denote by A the corresponding set of allocations.

An allocation strategy α maps each history of returns

$$h_t = (r_s)_{s\in\{0,\cdots,t-1\}} \in \mathcal{H}$$

to an allocation $\alpha(h_t) \in A$. We denote by $\mathcal{A}$ the set of possible allocation strategies. For simplicity, our theoretical section assumes that trading costs are equal to zero, while we allow for realistic trading costs in the empirical section. The returns r^α_t associated to allocation strategy α in period t take the form

$$r_t^\alpha \equiv \sum_{i=0}^{I} \alpha^i(h_t) r_t^i.$$

The growth optimal portfolio. We consider the problem of an investor that seeks to maximize the expected long-run growth of her portfolio

$$\sum_{t=1}^{N} \log(1 + r_t^\alpha).$$

Under the standard Bayesian framework, the investor is equipped with a prior $\mu \in \Delta(M^N)$ over sequences of returns $(r_t)_{t\in\{1,\cdots,N\}}$. This prior captures the investor's beliefs over the possible evolution of returns. This may include the presence of positive auto-correlation in returns, the possibility that a bubble may be underway, and so on. The investor chooses the allocation strategy α that solves

$$\max_{\alpha\in\mathcal{A}} \mathbb{E}_\mu \left[\sum_{t=1}^{N} \log(1 + r_t^\alpha) \right]. \tag{1}$$

The difficulty we confront in this paper is that forming beliefs μ is difficult, especially when there is little data to inform the decision-maker. An indicator of this difficulty is that different well-informed investors will frequently disagree about expected market behavior. Beliefs over innovative assets, such as crypto-currencies, are inherently subjective. As a result, a given investor's belief may be thoroughly unrelated to the true process followed by returns. How should an investor aware of this difficulty approach asset allocation?

2.2 A well calibrated investor should have low drawdowns

We now establish a reference property that well-constructed portfolios must satisfy: if the investor has correct beliefs over the possible distribution of returns, her portfolio returns should experience low drawdowns with very high probability.

For any asset $i \in \{0, \cdots, I\}$ let e_i denote the allocation that places a weight of 1 on asset i. Take as given $\nu \in [0,1]$. For any allocation a, define the allocation

$$a^{\to i} = \nu e_i + (1-\nu)a$$

that shifts mass ν of the portfolio towards asset i. Similarly, given a strategy α, we denote by $\alpha^{\to i}$ the strategy that shifts a mass ν of the portfolio generated by α to asset i

Definition 1 (drawdowns)**.** *Given a realized sequence of returns $\mathbf{r} = (r_t)_{t\in\{1,\cdots,N\}}$, the relative drawdown of allocation strategy α relative to asset i is given by:*

$$\mathcal{D}_i(\alpha, \mathbf{r}) = \max_{T' \le T \le N} \sum_{t=T'}^{T} \log\left(1 + r_t^{\alpha^{\to i}}\right) - \log\left(1 + r_t^{\alpha}\right).$$

A special case of interest is that where $\nu = 1$. In that case, drawdown $\mathcal{D}_0(\alpha, \mathbf{r})$ corresponds to the usual drawdown against the safe asset, i.e. peak-to-trough losses against the safe asset. More generally drawdowns can be interpreted as a sample version of optimality conditions: they correspond to the maximum potential gains from adjusting a benchmark portfolio α towards a particular underlying asset i over each time interval $\{T', \cdots, T\}$.

Theorem 1 (well-calibrated investors experience low drawdowns)**.** *Consider the optimal allocation strategy α^* of a Bayesian decision maker solving investment problem* (1) *under a prior μ. If the investor has correct, priors, that is, if returns $\mathbf{r}$ are indeed distributed according to μ, then for all $\epsilon > 0$ and all $i \in \{0, \cdots, I\}$*

$$\lim_{N\to\infty} \text{prob}_\mu \left(\mathcal{D}_i(\alpha, \mathbf{r}) \ge \epsilon N\right) = 0.$$

As the proof shows (see Appendix A), the speed of convergence is related to the magnitude $\overline{r}$ of extreme daily returns. As the maximum amplitude of daily returns $\overline{r}$ grows large, it becomes more likely that portfolio returns experience high drawdowns.

2.3 MinMax Drawdown Control

An implication of Theorem 1 is that if a decision-maker experiences high drawdowns, then, her beliefs are likely misspecified. This makes low-drawdown portfolios an agnostic benchmark that investors seeking to invest in crypto-currencies or other innovative assets can build on.

Chassang (2018) characterizes the asset allocation strategies α which guarantee the best possible drawdown guarantees against arbitrary sequences or returns $\mathbf{r}$. Formally, these are the strategies that solve

$$\min_{\alpha \in \mathcal{A}} \max_{\mathbf{r} \in M^N} \max_{i \in \{0, \cdots, I\}} \lambda_i \mathcal{D}_i(\alpha, \mathbf{r}) \tag{2}$$

where $\lambda_0 = 1$ and $(\lambda_i)_{i \in \{1, \cdots, I\}} \geq 0$ parameterize the relative importance of different drawdowns for the investor. Setting maximum drawdown targets against the safe asset will pin-down parameters $(\lambda_i)_{i \in \{1, \cdots, I\}}$. Chassang (2018) also establishes that worst-case drawdowns under minmax drawdown strategies are of order $\sqrt{N}$.

The two asset case. Figure 2 shows that in the case of two reference assets — a risk-free asset 0, and a risky asset 1 — minmax drawdowns $(\mathcal{D}_0, \mathcal{D}_1)$ map out an intuitive two-dimensional frontier: $\mathcal{D}_0$ captures losses against the safe asset; $\mathcal{D}_1$ captures foregone performance relative to the risky asset. For every guaranteed maximum drawdown $\mathcal{D}_0$ against the safe asset, the frontier associates the best possible drawdown guarantee $\mathcal{D}_1$ for drawdowns against the risky asset.

This lets the decision-maker express preferences over risk without refering to a prior. A particularly risk-averse investor will prefer very low drawdowns with respect to the safe asset at the expense of higher drawdowns against the risky asset. Inversely, an aggressive investor will prefer low drawdowns against the risky asset, at the expense of higher drawdowns against

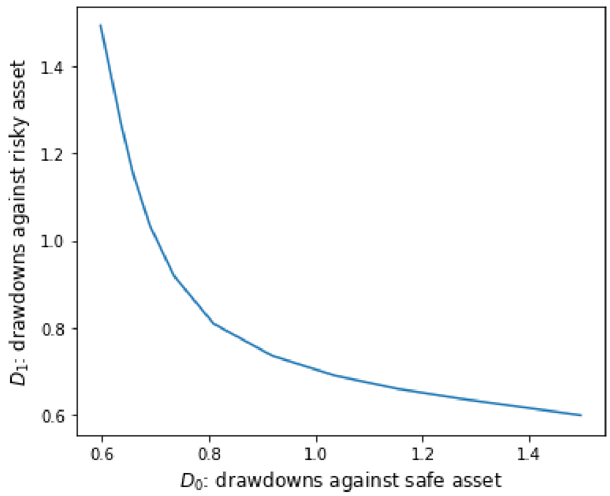


Figure 2: minmax drawdown frontier: $r^1 \in \{-.05, 0, .05\}, N = 252, \nu = 1$.

the safe asset.

It turns out that the optimal asset allocation strategy α_T^* at time T depends only on regrets $\mathcal{R}_{i,T} = \max_{T' \leq T} \sum_{t=T'}^{T} \log\left(1 + r_t^{\alpha \to i}\right) - \log(1 + r_t^{\alpha})$ for $i \in \{0, 1\}$. As Figure 3 illustrates, allocation weight $\alpha_{0,T}^*(\mathcal{R}_{0,T}, \mathcal{R}_{1,T})$ to the safe asset must be increasing in $\mathcal{R}_{0,T}$ and decreasing in $\mathcal{R}_{1,T}$.

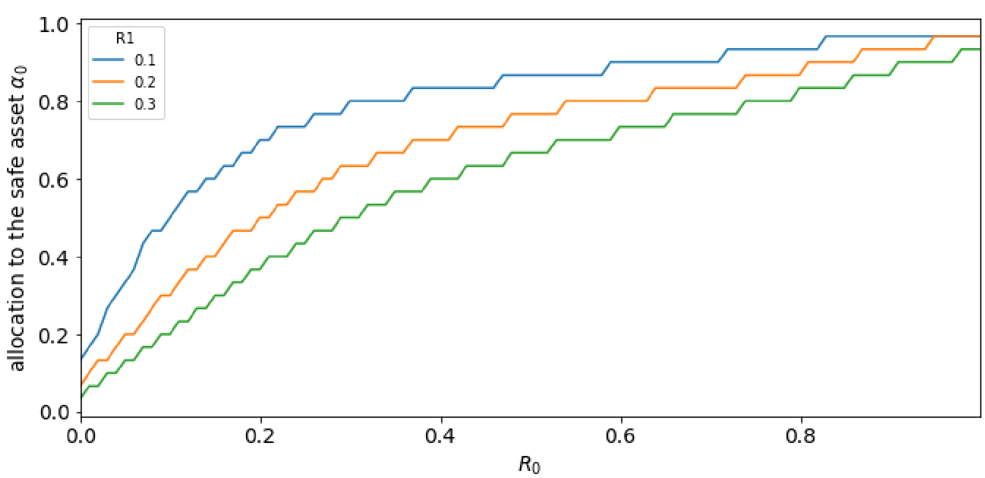


Figure 3: discretized policy functions as a function of $\mathcal{R}_0$ and $\mathcal{R}_1$ at time T=30.

Averaging over investors. Investment problem (1) considers the problem of an agent with a given investment time-frame $\{1, \cdots, N\}$. In practice, one may be interested in constructing long-term portfolio indices that are attractive to overlapping generations of investors. If investors each have a horizon of N periods, in any period $T \in \mathbb{N}$, investors with investment initiation dates

$$T - N + 1, T - N + 2, \cdots, T$$

are active. Let K_t denote the set of investors active at date t indexed by their initial investment date. Let α_k^* denote the MinMax Drawdown strategy corresponding to an investor with initial investment date $k \in K_t$.

A natural portfolio aggregation strategy is simply to average out the minmax drawdown portfolios α_k^* of active investors. At date t, the corresponding averaged portfolio $\overline{\alpha}_t^*$ takes the form

$$\overline{\alpha}_t^* \equiv \frac{1}{N} \sum_{k \in K_t} \alpha_{k,t}^*.$$

Importantly, averaged-out portfolios continue to provide drawdown guarantees.

Theorem 2 (low drawdown indices)**.** *Set $\nu = 1$. For all assets $i \in \{0, \cdots, I\}$, all infinite sequences of returns $(r_t)_{t \in \mathbb{N}}$, and all T', T such that $0 \leq T - T' \leq N$,*

$$\sum_{t=T'}^{T} \log(1 + r_t^i) - \log(1 + r_t^{\overline{\alpha}^*}) = o(N).$$

As a result, averaged-out low-drawdown strategies are well suited to form the basis of portfolio indices suitable for a broad class of investors. We now apply these strategies to baskets of crypto-currencies.

3 Application to Crypto-Currencies

Minmax Drawdown Control strategies are designed to provide robust performance guarantees in adversarial market environments. This section applies our framework to historical price data for leading crypto-currencies.

The risk-parity benchmark. In order to provide a comparison point, we include the performance of a risky-parity allocation strategy as a comparison. We set the volatility target of the risk-parity strategy so that it experiences the same drawdowns $\mathcal{D}_0$ against the safe asset as the MinMax Drawdown portfolio.

Formally, risk-parity is a particular implementation of modern portfolio theory. It assigns weights to risky assets that are inversely proportional to each asset's volatility:

$$\alpha_t^i = \rho \frac{1}{\widehat{\sigma}_{i,t}}$$

where $\widehat{\sigma}_{i,t}$ is a volatility estimate for asset i at time t, and ρ is a scaling parameter used to adjust overall portfolio volatility. Risk-parity implicitly assumes that expected returns are proportional to volatility. As a result, it fails to provide drawdown-control guarantees against either the safe or the risky asset: it can have too much volatility exposure in difficult times, and too little volatility exposure in good times.

3.1 Single asset risk-management: the case of bitcoin

We begin by illustrating the behavior of MinMax Drawdown portfolios in the two-asset case, using cash and bitcoin as reference assets. We consider investors with a one year horizon, who dislike drawdowns against the safe asset twice as much as drawdowns against the risky asset, i.e. we set $\lambda_0 = 1$ and $\lambda_1 = .5$. We allow for daily price movements in the range $M = [-5\%, +5\%]$. To reflect the need for plausible liquidity, we use January 1^{st} 2011 as the initiation date of the bitcoin portfolio. We use trading costs of .5%, but our results are essentially unchanged for trading costs of 1%. Given bitcoin's volatility, trading costs of that magnitude are not a limiting factor. In the case of a single risky asset, risk parity boils down to a volatility-control strategy.

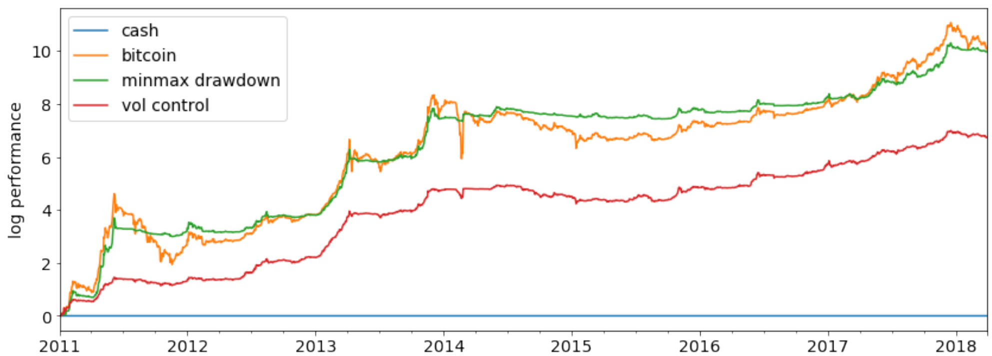


Figure 4: performance of minmax drawdown and volatility control portfolios.

As Table 1 and Figure 4 illustrate, both volatility control and MinMax Drawdown Control considerably improve the risk-reward trade-off of investing in bitcoin as captured by the Sharpe ratio. However, in order to match the realized drawdowns $\mathcal{D}_0$ of MinMax Drawdown Control, volatility control must considerably reduce its exposure throughout the investment period, which significantly reduces the corresponding annualized returns. As a result, MinMax Drawdown Control achieves a much better performance-to-drawdown ratio than volatility control.

	minmax drawdown	volatility control	bitcoin
Sharpe	4.14	3.59	1.75
annualized returns	2.96	1.53	3.01
drawdown $\mathcal{D}_0$	0.50	0.50	0.93
volatility	0.71	0.42	1.71

Table 1: performance metrics for minmax drawdown control, volatility control, bitcoin.

3.2 Multi-asset crypto-currency portfolios

Figure 5: bitcoin, ethereum and ripple over our selected sample period.

We now turn to the case of multiple risky assets and consider a portfolio allocation problem in which the investor seeks to optimize over cash, bitcoin, ethereum and ripple. To reflect the need for sufficient liquidity, we use January 1^{st} 2016 as the first date of inclusion of etherum and ripple as underlying assets.

	minmax drawdown	risk parity	equal-weight
Sharpe	7.34	4.02	2.93
annualized returns	6.88	2.11	5.17
drawdown $\mathcal{D}_0$	0.60	0.60	0.93
volatility	0.93	0.52	1.76

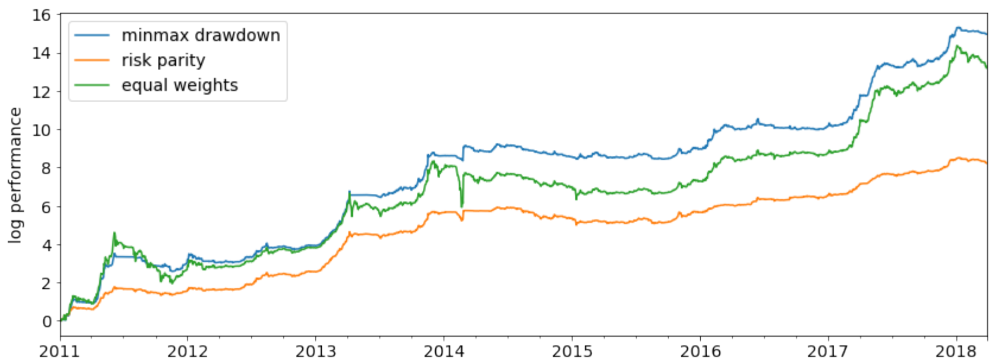
(a) performance since 2011

	minmax drawdown	risk parity	equal-weight
Sharpe	15.22	5.90	12.73
annualized returns	12.68	2.18	13.67
drawdown $\mathcal{D}_0$	0.43	0.26	0.70
volatility	0.83	0.37	1.07

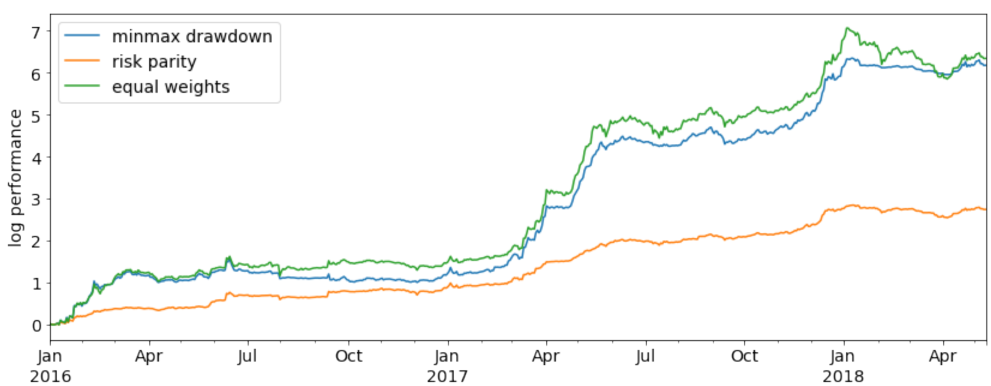
(b) performance since 2016

Table 2: performance metrics for minmax drawdown control, risk-parity, and equal-weight portfolios, starting from 2011 and 2016.

Again, both risk-parity and MinMax Drawdown Control improve on the Sharpe ratio achieved by a regularly rebalanced equal-weight portfolio. However, MinMax Drawdown Control achieves much higher annualized returns than risk-parity for the same worst-case drawdown. In fact, by successfully adjusting its exposure toward the best performing underlying crypto-currency, MinMax Drawdown Control achieves better raw performance than the much riskier equal-weight portfolio.



(a) performance since 2011



(b) performance since 2016

Figure 6: performance of minmax drawdown control, risk-parity, and equal weight portfolios, starting from 2011 and 2016.

4 Conclusion

A well calibrated Bayesian investor — i.e. an investor whose prior belief over the underlying stochastic process for returns is correct — should not experience large drawdowns. An agnostic investor can still achieve low worst-case drawdowns by following a MinMax Drawdown Control strategy. Importantly, such strategies can be aggregated to form low-drawdown portfolio indices, suitable for overlapping generations of investors. Both theoretically and empirically, the approach is well suited to guide portfolio allocation over innovative asset classes such as crypto-currencies.

Appendix

A Proofs

Proof of Theorem 1: The proof relies on the fact that having low drawdowns is a sample expression of optimality conditions. By definition, at any history h_t of returns before time t, the investor's optimal allocation $\alpha^*(h_t)$ cannot be improved by shifting the allocation towards any single asset allocation e_i. Hence, for all $i \in \{0, \cdots, I\}$, and all histories h_t

$$\mathbb{E}_\mu \left[\log\left(1 + r_t^{\alpha \to i}\right) |h_t\right] - \mathbb{E}_\mu[\log(1 + r_t^\alpha)|h_t] \leq 0$$

Let $m = \log(1 + \overline{r}) - \log(1 - \overline{r})$ denote the maximum per-period difference in log returns. Using the Azuma-Hoeffding inequality (see Cesa-Bianchi and Lugosi (2006), Lemma A.7 for a reference) this implies that when returns are drawn from the investor's prior, then, for all $T' \leq T \leq N$, for all i

$$\text{prob}_\mu \left(\sum_{t=T'}^{T} \log\left(1 + r_t^{\alpha \to i}\right) - \log(1 + r_t^\alpha) \geq \epsilon N \right) \leq \exp\left(-\epsilon^2 N / 2m^2\right).$$

Using the union bound it follows that $\text{prob}_\mu(\mathcal{D}_i(\alpha^*, \mathbf{r}) \geq \epsilon N) \leq N^2 \exp(-\epsilon^2 N/2m^2) \to 0$, which concludes the proof. ■

Proof of Theorem 2: Let $\overline{K} = \cup_{t \in \{T', \cdots, T\}} K_t$. It corresponds to the set of investors who may be active at some period in the interval $\{T', \cdots, T\}$. We have that $\mathsf{card}\ \overline{K} \leq 2N$. By Jensen's inequality, for any $T' \leq T$, we have that

$$\begin{aligned}\sum_{t=T'}^{T} \log(1+r_t^i) - \log(1+r_t^{\overline{\alpha}^*}) &\leq \sum_{t=T'}^{T} \log(1+r_t^i) - \frac{1}{N}\sum_{k\in K_t} \log(1+r_t^{\alpha_k}) \\ &\leq \frac{1}{N}\sum_{k\in\overline{K}} \max_{T'\leq T_0\leq T_1\leq T} \sum_{t=T_0}^{T_1} \log(1+r_t^i) - \log(1+r_t^{\alpha_k}) \\ &\leq O(\sqrt{N})\end{aligned}$$

where the last inequality follows from the fact that each underlying MinMax Drawdown strategy ensure that drawdowns are uniformly bounded, and of order $\sqrt{N}$. ■

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